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Original Research Article

Fractal Analysis of the Asma al-Ḥusna Mo'aqqali Inscription at the Pir Bakran Shrine, Isfahan

Keyhane Reisi[1]; **Jalil Jokar**[2]

1. Master's degree in Persian Painting, The Faculty of Handicrafts, Isfahan University of Art, Isfahan, Iran.
2. Corresponding author, faculty member of the Islamic Art Department, Faculty of Handicrafts, Isfahan University of Art, Isfahan, Iran.
E-mail: j.jokar@aui.ac.ir

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Abstract

Introduction: The Asma al-Husna inscription at the Pir Bakran Shrine is a notable example of Islamic art from the Ilkhanid period, as a manifestation of the divine names and a masterpiece of Islamic calligraphy. In terms of scale and word count, it is among the largest Mo'aqqali inscriptions in the history of architectural ornamentation in Iran. Its existence signifies the glorification and sanctification of God through art. This study applies fractal analysis to examining the complex and repetitive patterns in the inscription. The research seeks to answer the main question: How can the Asma al-Husna inscription at the Pir Bakran shrine be analyzed based on fractal geometry? In this regard, answering two subsidiary questions is essential: How is self-similarity manifested in the structure of the inscription? How can the fractal dimension of the inscription be calculated?

Research Method: This research is fundamental in its aim and descriptive-analytical in its methodology. Data collection was conducted through fieldwork and documentary research. Data analysis and fractal dimension calculation were performed using the Fractalyse software through simulation methods and the extension of filled and empty spaces, considered as a spatial fractal.

Findings: The structure of the inscription displays self-similarity across multiple layers, including the initial checkerboard pattern, the calligraphic baseline, the geometric design, and its components. The fractal dimension of the inscription was calculated to be 2.12 (decimal). The repetition of divine names within intricate patterns reflects divine order and the concept of infinity. Furthermore, fractal dimension calculations of the Mo'aqqali scripts in the Asma al-Husna inscription, compared with those of the opposing inscription (Salawat Kabira), revealed identical density, distribution of filled and empty spaces, and fractal breaks in both inscriptions. The arrangement of words in both inscriptions appears to be meticulously calculated.

Conclusion: Based on fractal analysis, the degree of conformity of the inscription's geometric structure with non-Euclidean fractal geometry (the geometry of nature) is determined, quantifying the complexity, roughness, and porosity of the inscription. The inscription exhibits self-similarity in its structure and arrangement of words, as well as a fractional dimension of 2.12, indicating adherence to non-Euclidean fractal geometry. Moreover, the high complexity of the script (1.762), in harmony with the opposite inscription, attracts and engages the viewer's effort to read the inscription.

Keywords

Pir Bakran Shrine, Asma al-Husna Inscription, Mo'aqqali script, Fractal Geometry, Self-Similarity, Fractional Dimension

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Introduction and Problem Definition

Fractal geometry is a modern geometry that, even unconsciously, has been employed by Iranian artists in architectural ornamentation through inspiration from nature. Furthermore, Islamic art is profoundly intertwinedwith spiritual, philosophical, and mystical concepts. Many Islamic geometric patterns symbolize divine order and infinity —embodying the notion of unity in multiplicity and multiplicity in unity, which closely resonates with the concept of fractals (the similarity of the part to the whole). Therefore, traces of fractal geometry can be sought in traditional artworks. The Divine Names (Asma al-Husna) have long attracted the attention of Islamic thinkers and artists and have been used in architectural inscriptions in different forms. The Pir Bakran Shrine, a masterpiece of the Ilkhanid era, not only contains exquisite architectural ornaments but also features a prominent inscription bearing the Asma al-Husna written in Mo'aqqali script. This inscription stands as a remarkable example of the combination of art and religion in the history of Iranian architecture due to its artistic and spiritual value. The inscription is significant not only for its calligraphy and architectural decoration but also for being a religious manifestation of the Divine Attributes in Islamic art. The widespread use of Mo'aqqali inscriptions from the medieval Islamic period to the present has made this branch of Islamic art, which also has a mysterious aspect not easily readable by everyone, very attractive and significant. Therefore, conducting research on these inscriptions and their precise interpretation, along with geometric analysis of their formal structure, are essential to reveal new dimensions of these undiscovered works and to provide a scientific framework for designing and analyzing such inscriptions for the audience. Since most previous studies have emphasized the emotional aspects of art and paid less attention to its rational dimensions, an interdisciplinary approach can disclose the hidden layers of artworks and reveal the connection between fractal order and mathematical order within them. Unfortunately, no research has been conducted on these inscriptions from the perspective of fractal non-Euclidean geometry. Therefore, this study aims to take a modest step in this direction. This research seeks to analyze the Asma al-Husna inscription of the Pir Bakran shrine from the viewpoint of fractal non-Euclidean geometry and examine the complex and repetitive patterns in the inscription's structure. The main question addressed is: How can the Asma al-Husna inscription at the Pir Bakran shrine be analyzed based on fractal geometry? In this regard, answering two subsidiary questions is essential: How is the feature of self-similarity manifested in the structure of the inscription? How can the fractal dimension of the inscription be calculated? To this end, the study initially introduces the concepts and research framework, and then describes and analyzes the inscription. Crucially, the fractal analysis of the aforementioned inscription—including assessment of self-similarity and computation of fractal dimensions—necessitates computational and software-based methodologies. In the present research, these analytical procedures have been rigorously and comprehensively applied to the subject inscription through advanced quantitative techniques.

Research Method

This research is fundamental in terms of its objective, and employs a descriptive and analytical methodology. Data collection was conducted through fieldwork using direct observation and photography. supplemented by documentary (library) sources to establish theoretical foundations. . For analysis, the study first describes the structure and theoretical basis of fractal geometry as well as the structure of the inscription. It then demonstrates the presence of non-Euclidean (fractal) geometry within. the inscription. Specifically, after

drawing the patterns using AutoCAD software, their self-similarity is examined. Afterwards. These patterns are subsequently converted into shapefile[1] format and imported as vector files into Fractalyse software, where their fractal dimension is calculated using the box-counting method and logarithmic formula. Furthermore, through three-dimensional modeling of the inscription lines in 3ds Max software, the study simulates and generalizes the filled and empty spaces as a spatial fractal (porous cube). By applying the relevant formula, the fractal dimension of the inscription is computed. The obtained data are subjected to qualitative and quantitative fractal analysis in order to examine the complex and repetitive patterns within the inscription.

Research Background

Several studies have been conducted on the Asma al-Husna inscription of the Pir Bakran shrine. For instance, «Hamzavi and Aslani» (2012), in their book «Architectural Decorations of the Pir Bakran Shrine», first examined the Pir Bakran district, its historical background, and climatic characteristics. They then introduced the architectural spaces of the shrine and its functions in the past and present. The introduction of «Mohammad ibn Bakran» and the study of architectural ornaments based on their placement in various spatial contexts are among the other focal points of this research. «Honarfar» (1971), in his book «The Treasury of Isfahan's Historical Monuments», presented an introduction tothe Pir Bakran shrine and its various sections, and also introduced and transcribed the Asma al-Husna inscription of the shrine. Fractal geometry has been explored from various perspectives in traditional architecture and its ornamentation by researchers. «Reisi et al.» (2024), in their article «An Analysis of Fractal Geometry in the Mo'aqqali Tilework of the Spandrels of the Northern Iwan in Hakim Mosque in Isfahan», studied the tilework patterns from a fractal perspective. Their research, after calculating the fractal dimension of the motifs by two methods—the logarithmic box-counting method and software analysis—showed that these geometric patterns possess fractional dimensions, with only minor differences between the two methods, in full conformity with fractal dimensions. The self-similarity property in the motifs was also examined and elucidated. «Mirian» (2011), in the article «The Role of Fractals in Geometry, Mathematics, and Their Relation to Islamic Patterns in Iranian Monuments and Mosques», studied the similarities between fractal geometry and Islamic patterns, such as self-similarity, attention to intricate details, repetition of unit cells, and adherence to a replication rule. «Belilan Asl et al.» (2011), in their article «Examining the Geometric Features of Knots in Islamic Decorations from the Perspective of Fractal Geometry»,compared the geometric structure of knots with fractal geometry and discussed their common features. «Beigmohammadi and Panahi» (2017), in their article «Investigating Order in Iranian Islamic Architecture with a Fractal Geometry Approach in Decorations (Case Study: Sheikh Lotfollah Dome)», simulated the interior surface beneath the dome of the Sheikh Lotfollah Mosque using Rhino software. By comparing it to the sunflower pattern and examining the feature of self-similarity, they concluded that the dome exhibits fractal characteristics consistent with natural patterns found in nature. «Rezazade» (2020), in his article «A Visual and Structural Analysis of Fractal Geometry in the Decorations of Sheikh Lotfollah Mosque», examined fractal characteristics (self-similarity) in the mosque's decorations, such as moqarnas and the patterns beneath the dome. The author concluded that the traditional architect, through mathematics and proportions, created concepts such as «unity in multiplicity» in architecture, all of which are inherently compatible with fractal forms. «Ismail Attia» (2020), in his article «Fractal Forms in Islamic Design», studied fractal geometry in the

Sultan Hassan Mosque in Egypt. This research indicated that interior design using Islamic fractal motifs positively affects the health of its occupants, as it reduces stress levels, increases relaxation, and improves long-term memory. «Abdelsalam and Ibrahim» (2018), in their article «Fractal Dimensions of Islamic Architecture», analyzed fractal geometry in the decorations of the Sultan Hassan Madrasa. This study analyzed the plan, elevation, and decorations of the school from the perspective of fractal geometry and golden ratios, ultimately investigating the overlapping spaces in the school's plan by altering parameters in Grasshopper software—a plugin for Rhino that facilitates parametric modeling. The research emphasizes the existence of fractal geometric features in the primary geometry, which is formed by the repetition of the building's plan. «Khamjane and Benslimane» (2017), in their article «The Golden Mean, Fractals, and Islamic Geometric Patterns», identified some aesthetic aspects related to mathematical properties such as the golden ratio and fractal geometry in the Islamic art geometry of the Attarine Madrasa (Fes, Morocco). Despite the aforesaid studies on fractal geometry in Islamic architecture, no research has yet focused specifically on fractal geometry in the Asma al-Husna inscription of the Pir Bakran shrine. Previous studies have only introduced this inscription superficially. Among the innovations of this research is the calculation of the fractal dimension of the inscription based on simulation and the generalization of filled and empty spaces as a spatial fractal, as well as the use of the fractal dimension of the inscription's script to quantify its complexity. This allows access to hidden layers, the existing mathematical and geometric order within the inscription, and an examination of the effects of this complexity on the viewer.

Theoretical Structure

1. Mo'aqqali Script: This script is considered by some scholars as a type of Kufic script. Mo'aqqali was commonly used in tilework and architectural decoration but was not typically employed in manuscript writing on paper, as it is entirely composed of straight-line strokes and exhibits the most geometric proportion among all scripts (in calligraphy, sath and dawr are fundamental principles, referring to straight lines and curved or circular strokes, respectively). In Mo'aqqali script, the thickness of all letters is uniform, and it is regarded as the most balanced script in terms of positive and negative space (density and openness). Initially, Mo'aqqali was designed on graph paper and then used for architectural ornamentation (Ghelichkhani, 2011, 355). It is typically structured within squares, rectangles, and other geometric shapes. The rectangular and sometimes square shapes of the letters distinguish it from other scripts; however, the precision in selecting the angles of these rectangles or squares results in a completely harmonious geometric composition. This script is often devoid of punctuation or vowel notations (Faza'eli, 2011, 160).

2. Fractal Geometry: Fractal science is a new perspective in the geometry of natural reality, proposed by the French mathematician Benoît Mandelbrot. This geometry studies the geometric aspects of natural structures. According to Mandelbrot, the world has fractal dimensions, and natural structures possess fractal properties. A fractal describes a shape, pattern, or process that is self-similar (Ziukelis et al., 2022). In fact, fractal geometry studies mathematical images that represent natural forms. They provide an endless chain of details that are self-similar and highly intricate. Self-similarity is a defining feature of fractal patterns (Ashraf Ganjoui & Iranmanesh, 2024). According to the principle of self-similarity, a fractal generates self-similar structures at different scales, creating a hierarchy of connections in subsequent stages (Ghouchani & Mokaberian, 2022). For a shape to be self-similar, it must be divisible into parts that are smaller copies resembling the whole shape to some extent (Aswathy, 2016). Fractional dimensionality is another important feature of

fractal geometry. This geometry is used to describe complex and irregular lines, surfaces, and volumes that have dimensions between the integer Euclidean dimensions (Mandelbrot, 1982, 44). The fractal dimension serves as a quantitative measure for attributes such as irregularity, complexity, and roughness, effectively providing a mathematical basis for these qualitative descriptors (Lesmoir-Gordon, 2010, 4).

3. The importance of calculating fractal dimension: Although the concept of fractals initially emerged in mathematics and natural sciences, it also plays an important role in art. Since the human mind inherently possesses a fractal structure, detecting rhythms and repetitions is satisfying for viewers, making fractal characteristics essential in decorative design (Salingaros, 2022, 181). Fractal patterns are visually preferred by viewers (Nasehi et al., 2024). Complex decorations significantly influence the quality of architectural spaces by enhancing their qualitative attributes and imparting identity to the space. This complexity introduces ambiguity to the decorations, and the greater the ambiguity, the more imagination and sense of wonder it evokes in the viewer (Mahdavi Pour et al., 2017). Highly intricate decorations engage the viewer, encouraging prolonged contemplation. The viewer unconsciously spends time exploring these complexities and extracting meaning from them (Salingaros, 2008, 149). Therefore, the degree of complexity in an artwork must be carefully considered. Fractal dimension quantifies the complexity, fragmentation, and distribution of filled and empty spaces, thereby aiding in a more precise analysis of an artwork.

4. Common Methods for Calculating Fractal Dimension: In fractal geometry, fractal dimension is a mathematical measure used to determine the degree of complexity of a displayed texture. The concept was first introduced by mathematician Felix Hausdorff in 1918 (McClure, 2022, 42). Unlike Euclidean shapes, which feature integer dimensions, fractal shapes possess fractional (non-integer) dimensions. Therefore, the fractal dimension of a horizontal sea line is equal to one, while a fully black plane has a fractal dimension of two. The presence of complexities, irregularities, and empty spaces creates fractional dimensions. The greater the complexity and fragmentation of an image, the higher its fractal dimension, approaching the value of two (Lesmoir-Gordon, 2010, 5) (Fig 1). Fractal dimension can be calculated using various methods, with two common approaches being the similarity dimension and the box-counting dimension.

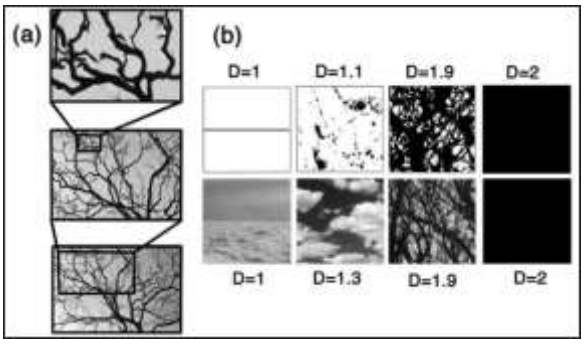


Fig 1. Calculation of fractal dimension at different scales. Source: Taylor, 2006.

4.1. similarity dimension: The similarity dimension represents the most straightforward method for calculating fractal dimension and is associated with fractals generated by iterative systems (Glass, 2011, 25). In all self-similar structures, there is a relationship between the scaling factor and the number of smaller divisions into which the original structure is divided. If N is the number of self-similar pieces and S is the scaling factor by which the original line is divided, then the fractal dimension D is given by the following

formula (Mandelbrot, 1982, 101) The similarity dimension has been calculated for the porous cube (Figure 2a) and the Cantor set (Fig 2b).

(1) $D= \text{Log } N/ \text{Log } S$

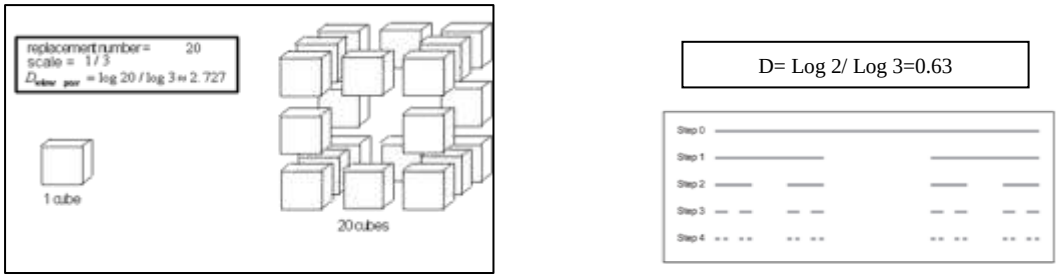


Fig 2. Similarity dimension. Reference: a. https://www.wahl.org/fe/HTML_version/link/FE4W/c4.htm and b. Source: Shirali, 2014.

4.2. Box-Counting Dimension: The box-counting method is one of the most commonly used techniques for calculating the fractal dimension. To apply this method, a grid of square boxes is overlaid on the image, with the size of each box denoted by S. The number of boxes N(S) that cover any part of the image is then counted. This count reflects the amount of positive and negative space of the pattern per unit area; in other words, empty (negative) spaces are also considered. Boxes that do not cover any part of the pattern create gaps on the surface, which cause the fractal dimension to deviate from the integer value of 2 and shift toward a number between 1 and 2. This process is repeated by by reducing S into finer grids and recounting the number of boxes N(s) that cover the image. The smaller the box size, the more accurate the calculated fractal dimension becomes. The number 1/s represents the number of boxes along one edge of the grid (Fig 3). Fractal dimension D is calculated via the following formula (Bovill, 1996):

(2) $$D_b=\frac{[\log(N(S2))-\log(N(S1))]}{[\log(\frac{1}{S2})-\log(\frac{1}{S1})]}$$

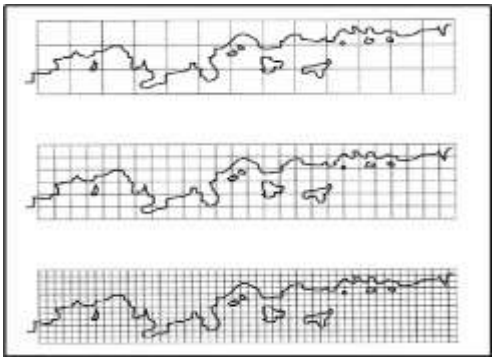


Fig 3. Box-counting grids. Source: Bovill, 1996.

The Pir Bakran Shrine

The Pir Bakran Shrine is located about 30 kilometers southwest of Isfahan, in the village of Pir Bakran within the Lenjan district. It dates back to the 8th century AH (703-712 AH) during the reign of Oljeitu (Fig 4). Pir Bakran (Mohammad) was a renowned figure, a scholar, and a mystic of the region, whose tomb is situated within this shrine, which also served as his teaching place. To honor his religious and scholarly status, the tomb is decorated with various stucco works and tiles typical of that era (Honarfar, 1971, 253). The old Jewish cemetery of Isfahan, known as Ester Khatoon, was also located nearby. This area was a major Jewish settlement, and its historical relics are attributed to Shushandukht (Queen of Susa), the Jewish queen of Yazdegerd I (Rafiei Mehrabadi, 1973, 863-865). The Shrine's architecture is similar to the Sasanian-era Taq Kasra (Iwan of Ctesiphon), indicating that the Sasanian architectural style was still influential at that time (Hamzavi & Moazeni, 2013). The building has three floors and features Ilkhanid architectural characteristics, including a porch (riwaq), a courtyard (sahn), an entrance portal (sardar), and the tomb chamber (Hamzavi & Aslani, 2012, 39).



Fig 4. General view of Pir bakran. Source: Shahsavarani et al., 2016, 3.

Introduction of the Study Subject: The Asma al-Husna Inscription

A plaster inscription in Mo'aqqali script on a checkered background, measuring 242 by 220 centimeters, inspired by a woven texture, is located opposite the inscription of «Salawat Kabira» on the western side of the Shrine's courtyard, mirroring the eastern side, which features two piers and a central soffit. This inscription is adorned with the Asma al-Husna (the Beautiful Names of God) (Honarfar, 1971, 260; Hamzavi & Aslani, 2012) (Fig 5).

بسم الله الرحمن الرحیم هوالله الذی لا اله الا هو الرحمن الرحیم الملک القدوس السلام المومن المهیمن العزیز الجبار الملک الخالق البارى المصور الغفار القهار الوهاب الرزاق الفتاح العلیم القابض الباسط الخافظ الرافع المعز المذل السمیع البصیر الحکیم العدل اللطیف الخبیر الحلیم العظیم الغفور الشکور الحفیظ المقیت الحسیب الجلیل الکریم الرقیب المجیب الملمع الحکیم(تکرار دوم) الودود المجید الباعث الشهید الحق الوکیل القوى المتین الولى الحمید المحصى المبدى المعید المحیى الممیت الحى.

In the name of God, the Most Gracious, the Most Merciful. He is Allah— there is no deity except Him. He is the Most Gracious, the Most Merciful, the Sovereign Lord, the Most Holy, the Source of Peace, the Believer, the Vigilant, the Almighty, the Compeller, the Proud, the Creator, the Originator, the Fashioner, the Pardoner, the Dominator, the Bestower, the Dispenser of all good, the Victorious, the All-Knowing, the Withholder, the Expander, the Abaser, the Exalter, the Honorer, the Humiliator, the All-Hearing, the All-Seeing, the All-Wise, the Just, the Gentle, the All-Aware, the Forbearing, the Magnificent, the All-Forgiving, the Appreciative, the Preserver, the Sustainer, the Reckoner, the Majestic, the Generous, the Watchful, the Responsive, the Illuminator, the All-Wise (repeated), the Loving, the Glorious, the Resurrector, the Witness, the Truth, the Trustee,

the Strong, the Firm, the Protecting Friend, the Praiseworthy, the Enumerator, the Initiator, the Restorer, the Giver of Life, the Bringer of Death, the Ever-Living.
Directly opposite this inscription is the *Salawat al-Kabira* inscription of the Fourteen Infallibles, of the same dimensions, bearing the following text (Fig 6):
اللهم صل علی محمد المصطفی و صل علی علی المرتضی و صل علی فاطمۀ زهرا و صل علی خدیجۀ الکبرا و صل علی حسن المجتبا و صل علی حسین شهید کربلا و صل علی علی زین العابدین و صل علی محمد باقر و صل علی جعفر صادق و صل علی موسی الکاظم و صل علی علی موسی رضا و صل علی محمد تقی و صل علی نقی و صل علی حسن العسکری و صل علی محمد مهدی.
Translation: O Lord, bestow Your blessings and mercy upon Muhammad al-Mustafa and Ali al-Murtaza, upon Fatimah al-Zahra and Khadijah the Great, upon Hasan al-Mujtaba and Husayn, the Martyr of Karbala, and bestow blessings and mercy upon Zayn al-‘Abidin, upon Muhammad al-Baqir and Ja‘far al-Sadiq, upon Musa al-Kazim and Ali al-Rida, upon Muhammad al-Taqi, upon Ali al-Naqi, upon Hasan al-‘Askari, and upon Muhammad al-Mahdi.
The text of the inscription is derived from one of the talismans (hirz) of Imam Sajjad. The inscription begins with the name of the Prophet Muhammad and, symmetrically, concludes at its center with the name of Imam Mahdi (Muhammad). The execution of the inscription—in terms of proportions, consistency of script, alignment, and the maintenance of its checkered grid framework—shows less precision compared to the *Asma al-Husna* inscription (Rajai Baghsorkhi, 2021).



Fig 5. Asma al-Husna Inscription.
Source: Authors’ Archive.



Fig 6. Salavat Kabira inscription.
Source: Rajai Baghsorkhi, 2021.

1. Formal and Fractal Analysis of the Asma al-Husna Inscription: In this inscription, the names of God are written in Mo'aqqaliscript, beginning from the lower right corner and spiraling clockwise in a spiral motion (a movement starting from a single point and progressing clockwise toward the center) (Figs 6 and 7). The Divine names start with the phrase «Bismillah al-Rahman al-Rahim» and conclude with the name «Al-Hayy» (The Ever-Living) at the center. Including repeated names, a total of 66 Divine names are arranged side by side in this inscription, achieving perfect balance and harmony between positive and negative space.



Fig 6. Graphic design of the inscription of Asma al-Hasani. Source: Drawing: Authors.

2. Analysis of Self-Similarity Features in the Inscription: Considering that the inscription is written in Mo'aqqaliscript and the lines follow a checkered pattern, the initial design clearly shows the repetition of small squares within larger squares. Therefore, at the first stage of its creation—the checkered grid—the inscription exhibits self-similarity. By examining the baselines (Korsi lines), which form concentric squares, one can observe self-similarity within these nested squares. Moreover, the manifestation of unity in multiplicity and multiplicity in unity is reflected in the Divine names arranged in a clockwise spiral that leads to the word «Al-Hayy» (The Ever-Living) at the center (blue lines shown in Fig 7). The total of 66 divine names, symbolizing monotheism and corresponding to the Abjad[2] numerical value of the name «Allah», represents another definition of self-similarity. A detailed geometric analysis and drawing of the squares present in the design reveal overlapping squares of various sizes, indicating the inscription's self-persistence (Fig 8). On one hand, the components of the design are repeated at different scales and rotations. On the other hand, straight lines of varying lengths extend and break throughout the pattern, filling the space (Fig 9). These factors collectively demonstrate the presence of self-similarity in the inscription.

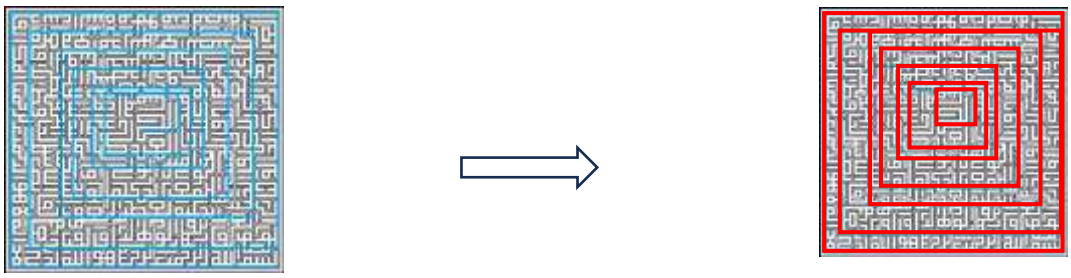


Fig 7. Graphic design of the inscription Asma al-Husna and the Korsi script. Source: Authors.

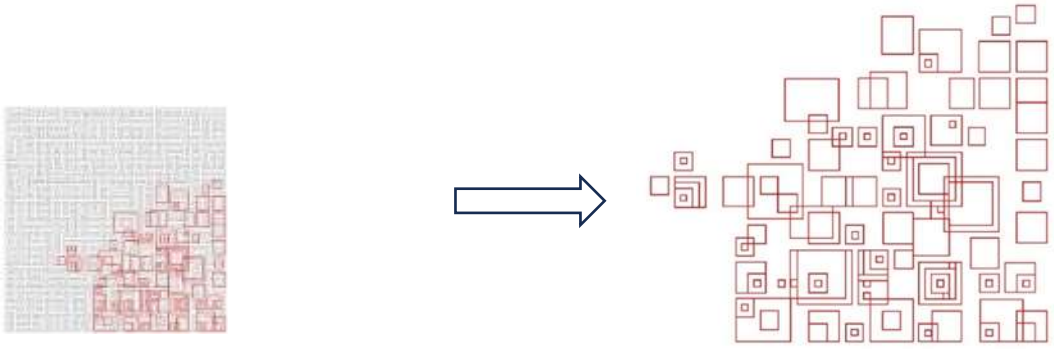


Fig 8. Self-similarity in repeating squares. Source: Authors.



Fig 9. Self-similarity in repeating elements. Source: Authors.

3. Calculation of the Fractal Dimension of the Asma al-Husna Inscription: To calculate the fractal dimension of the inscription, a cubic grid is employed, using the simulation and extension of filled and empty spaces as a spatial fractal, analogous to a porous cube shown in Figure 2. Considering that the inscription is approximately square with dimensions of about 2 meters, a grid of cubes with approximate dimensions of 1.5 centimeters is applied on it. The depth of the inscription's plaster letters is about 2 centimeters; therefore, two rows of cubes of this size cover parts of the inscription (Fig 10). The fractal dimension is then calculated using formula (1):

S=132 (Number of cubes at the bottom of the image)

N=32205 (Number of cubes covering the letters of the inscription)

$$\frac{log\ 32205}{log\ 132} = \frac{4.5}{2.12} = 2 \cdot 12$$

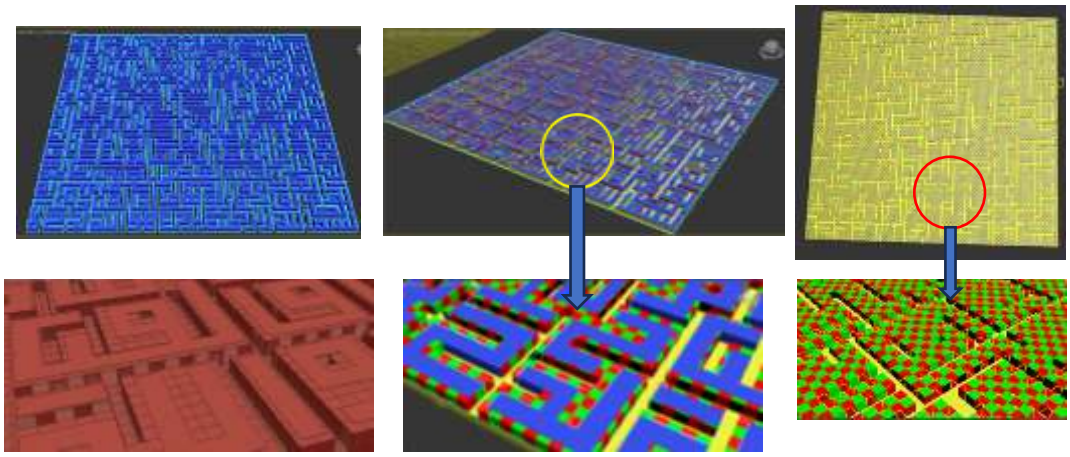


Fig 10. Cubes covering the letters of the inscription. Source: Drawing: Authors.

Based on the calculations, the fractal dimension of this inscription is 2.12 due to its roughness, empty spaces, and unevenness, which corresponds to the dimensions of fractal shapes. Furthermore, after linearizing the inscription's script, its fractal dimension was calculated using the box-counting method with the assistance of software. Since calculating the fractal dimension is a gradual process, for greater accuracy, the size of the squares must be gradually reduced to approximately fit the pattern. The software applied a five-stage procedure in which the squares were successively reduced in size and increased in number, so that by the fifth stage, the squares aligned closely with the design. As observed, the fractal dimension calculated at the fifth stage is the closest value to the dimension calculated by the software. The fractal dimension at each stage was calculated using Formula (2).

$$D(2-1) = \frac{log\ 49 - log\ 16}{log\ 7 - log\ 4} = 2.04$$

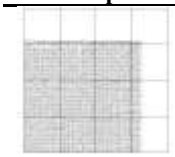
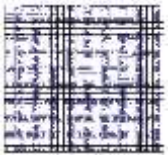
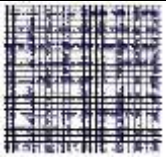
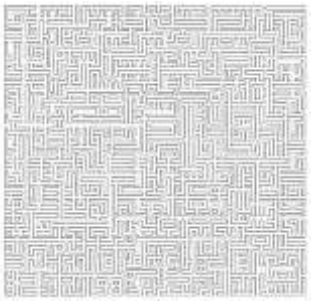
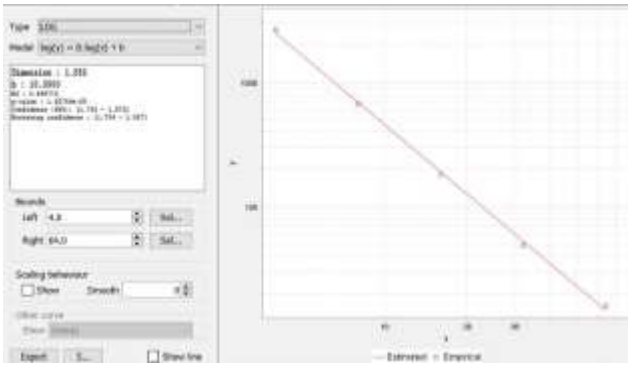
$$D(3-2) = \frac{\log 182 - \log 49}{log\ 14 - log\ 7} = 1.9$$

$$D(4-3) = \frac{log\ 675 - log\ 182}{log\ 27 - log\ 14} = 1.93$$

$$D(5-4) = \frac{log\ 2650 - log\ 675}{log\ 53 - log\ 27} = 1.89$$

The graph generated by the software includes five points with coordinates log (N) and log(s) at each stage. The slope of the line passing through these points represents the final fractal dimension, which is 1.852. Ultimately, the linearized design of the inscription also exhibits a fractional dimension (Table 1). Given the fractal dimension value (1.852), the linearized design of the inscription has a relatively high dimension, which indicates the high complexity of the inscription's pattern. Therefore, this artwork contains a high informational load for the viewer, leading to attraction, pause, and a sense of wonder. Initially, the viewer is drawn by the high complexity of the inscription and is subsequently stimulated to explore, attempt to read, and discover the inscription.

Table 1. Calculation of the dimension of the linear design of the inscription. Source: Authors.

Step 1	Step 2	Step 3	Step 4	Step 5
				
N=16 , S=4	N=49 , S=7	N=182 , S=14	N=675 , S=27	N=2650 , S=53
				
linearized design of Asma al-Husna inscription				
Dimension	1.852			

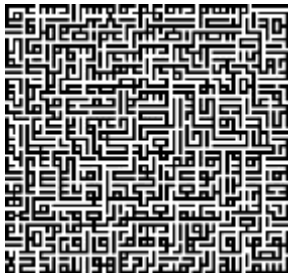
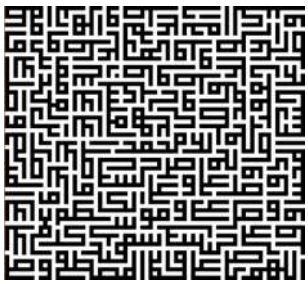
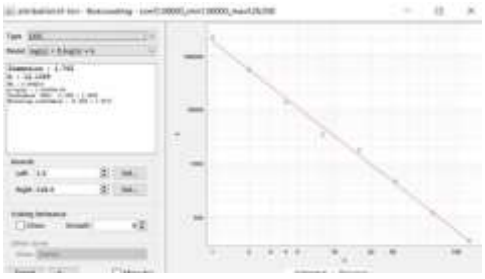
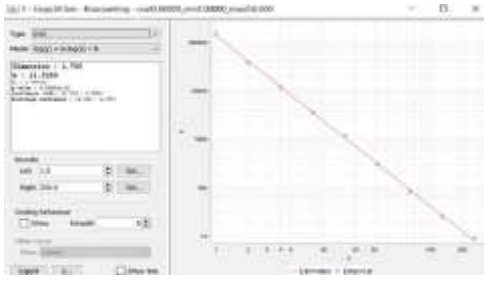
As observed, since the mentioned inscription possesses the feature of self-similarity and, simultaneously, has a fractional dimension, it can be confidently concluded that it embodies a fractal structure. In other words, its design corresponds closely to the geometry of nature.

Discussion

In this study, the Asma al-Husna inscription at the Pir Bakran shrine was examined from the perspective of non-Euclidean geometry. The arrangement of the names of God in concentric squares with clockwise and spiral rotation indicates the feature of self-similarity. Besides, due to the placement of the Mo'aqqaliscript on a checkerboard surface, repetition of squares at different scales is evident. The repetition of the inscription's components in various sizes, along with their rotation and fragmentation, further confirms the presence of self-similarity. The fractal dimension of the inscription is fractional due to the roughness and the presence of filled and empty spaces on its surface. By overlaying a cubic grid on the components of the inscription and counting them, the fractal dimension was calculated as 2.12, which corresponds to fractal dimensions in general. Additionally, the linearized design of the inscription was analyzed using software, and its fractal dimension was calculated by the box-counting method. The linear sample also exhibited a fractional dimension due to the fragmentation and empty spaces, indicating a high complexity of the inscription’s design. This high complexity captures the viewer’s attention, prompting pauses and encouraging them to examine and explore the inscription. Notably, after calculating the fractal dimension of the Salawat Kabira inscription, which faces the Asma al-Husna inscription, and comparing the two (given the structural similarity of the inscriptions, self-similarity in the Salawat al-Kabira was not separately examined), it was found that both inscriptions have very close fractal dimensions. The difference between their dimensions is minimal and negligible at 0.003 (Table 2). This indicates that the complexity, distribution of filled and empty spaces, and line density in both inscriptions are nearly identical. The artist-designer has carefully and harmoniously designed these two

inscriptions. One of the important features of traditional Islamic architectural decoration is the existence of symmetry amid asymmetry. The artist has intelligently respected and displayed this principle in these two inscriptions. As observed from the calculations, the minimal difference in the dimensions of the two opposing inscriptions indicates the equal use of positive and negative space in their design. In traditional design of motifs and inscriptions, balance and equilibrium between positive and negative space—which, from the perspective of Islamic art philosophy, represents a display of the perceived world and meaning—are of paramount importance. From an aesthetic standpoint, this balance and interaction between positive and negative spaces produce a sense of wonder arising from complexity, coupled with a perception of stability and strength. Moreover, the concept of imperfect symmetry, or symmetry amid asymmetry[3], is a feature derived from natural systems. In nature, what appears symmetrical at first glance often lacks perfect symmetry in its components. In this structure, although the two inscriptions appear symmetrical at first sight, a closer inspection reveals subtle differences that lead to a more precise reading of the inscriptions. Because the architectural structure of the iwan is symmetrical, the artist was required to design the two inscriptions in such a way that, despite textual differences, they visually maintain a decorative symmetry.

Table 2. Calculation of the writing dimension of the inscription of Salawat Kabira and Asma al-Husna Inscriptions. Source: Authors.

Asma al-Husna Inscriptions		**Salawat Kabira Inscription**	
			
			
Dimension	1.762	Dimension	1.759

The calculation of the fractal dimension of the inscriptions confirmed that, besides the fractional nature of the inscriptions' dimension, the arrangement of filled and empty spaces as well as the breaks was carried out in a thoroughly deliberate manner, demonstrating that Iranian artists of past centuries meticulously adhered to geometric and mathematical order in their works.

Conclusion

Based on modern fractal geometry, the degree of conformity of the inscription's geometric structure with non-Euclidean fractal geometry is determined. In other words, this geometry indicates the extent to which the formal structure of the inscription aligns with the geometry of nature. Through fractal analysis, the amount and type of complexity, filled and empty spaces, and roughness in artworks can be analyzed and quantified—an outcome that is impossible through non-fractal analysis. In the present study, fractal analysis of the Asma al-Husna inscription at the Pir Bakran shrine revealed that the inscription fully exhibits fractal characteristics, (self-similarity and fractional dimension). The Ilkhanid-period artist, trained with a refined sense in the artistic school of his era, created the work using repetitive and complex visual patterns and the principles of the Mo'aqqaliscript writing, without knowledge of non-Euclidean geometry. Notably, this research proves that the governing geometry of this inscription aligns with the geometry of nature, namely, fractal geometry. Moreover, the high complexity of the inscription enhances the quality and ambiguity of the space, as well as attracting and holding the viewer's attention. By comparing the complexity level (fractal dimension) of the Asma al-Husna inscription with that of the Salawat-e-Kabira inscription, it was revealed that the proportions of filled and empty spaces and the breaks in both inscriptions are approximately equal. This similarity results in the perception of symmetry between the two inscriptions at first glance, despite differences in their textual content. Therefore, the designer of these inscriptions could, in addition to their beautiful and geometric design, embedd mathematical symbols and themes within the inscriptions for thoughtful observers. These two inscriptions exemplify the coexistence of mathematical and geometric thinking alongside religious matters and art. Hence, ornamentation in this context transcends the mere provision of visual pleasure. Research of this kind, which uncovers subtleties, hidden aspects, and previously unexplored dimensions, offers audiences a deeper understanding of the art of past generations and provides guidance for contemporary artists in creating new works within traditional frameworks. Moreover, since traditional artists often relied on intuitive principles of design in producing motifs or inscriptions, the application of fractal analysis methods can serve as a valuable scientific and systematic approach for modern creators. This approach results in works that not only reflect the cultivated rationality and refined aesthetic sense of the artist but also embody the necessary scientific rigor and structural coherence.

Author Contributions

This article is the result of joint collaboration between both authors. The authors contributed to writing different sections, reviewing findings, analyzing results, and providing final approval of the manuscript.

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Appendix

1. A Shapefile is a simple, non-topological format used to store the geometric location and attribute information of a geographic feature. The geographic features within a Shapefile can be represented as points, lines, or polygonal areas.
2. The Abjad letters are a method for ordering letters in the Arabic language based on the original alphabet of the Phoenician script. The numbers assigned to the 28 letters of the «Abjad Kabir» range from one to one thousand. In fact, the numbers from 1 to 1000 are distributed among the 28 Arabic letters.
3. Perfect symmetry is only meaningful and definable within geometry. In a shape that possesses perfect symmetry—whether it is with respect to a line, an axis, or a point—the repeating parts around the axis or point of symmetry are exactly identical and are merely mirror reflections of each other. However, in imperfect symmetry, or symmetry within asymmetry—which is commonly found in nature—at first glance, the form appears symmetrical. That is, symmetry exists in the overall shape, but asymmetry prevails in the details. Examples include the leaves of trees or human faces.

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